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TWO-FLUID FLOWS OF A MIXTURE OF A GAS AND SOLID PARTICLES WITH "FILMS" AND "FILAMENTS" APPEARING IN FLOWS PAST IMPERMEABLE SURFACES*

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The two-fluid model with dispersed phase, representing a solid medium with zero intrinsic pressure, is used to study the appearance of "films" and "filaments" in the flow of a mixture of a gas and solid particles past impermeable surfaces. The films, endowed with surface properties (mass, etc.) and the filaments with analogous linear properties, first introduced in /1/ in the hydrodynamics of media without pressure with prohibited overtaking, appear in two-fluid models as a result of the intersection of dispersed-phase trajectories caused, for example, by inhomogeneities in the initial distributions /2—6/. In /7/ films were used to describe thin boundary layers of the solid phase forming at the walls and moving along them, and the possibility of the film separating (parting) from the wall is noted(**). The equations and conditions governing the formation, motion and separation of the films and filaments from surfaces are derived below, and various models of the detachment of films and their subsequent evolution are presented. It is shown how a detached (free) film collects particles, separating the regions containing a mixture from those of pure gas, and acting as if it were a filter. In the models with absorbent walls, where the particles terminate their existence, the same regions are separated by the tangential discontinuities of the particulate continuum through which pure gas flows. Cases are noted of the flows in nozzles where a film, after detaching itself, either becomes reattached to the wall or forms a filament on the axis. It is shows that particles may form zones of finite volume in flows past blunt bodies. The possibility of a film forming in a rarefaction flow, when a two-phase mixture flows supersonically past a sharp bend, is demonstrated.

1. In considering two-phase flows we shall limit ourselves to the two-fluid (two velocity and two-temperature) approximation. This means that all dispersed particles have, at every point, only one velocity and one temperature. In this approximation the particles, on being reflected from a rigid wall, inevitably find themselves within a boundary layer of zero thickness, and move with it along the surface of the body. This is how a surface film is formed with, generally speaking, infinite volume density ρ_s of the particles, and, as a rule, finite surface density ρ_s^σ. Here and henceforth the subscript s will indicate the parameters of the continuum of the particles (the "second" phase), and σ will refer to the film parameters. The gas parameters will be denoted by the same symbols without subscripts.

The appearance of a boundary film suggests, at first glance, that the model is imperfect, especially since the change to a three-fluid approximation with two particulate media (incident onto and reflected from the wall) yields what seems to be a more complete description without introducing the concept of a film. However, it so happens that when we stipulate the existence of particulate media moving with respect to each other, their collisions must be taken into account. If at the same time the mean free path λ of the reflected particles within the flow of the oncoming particles satisfies the inequality $\lambda \ll L$, where L is the characteristic dimension of the flow, the replacement of the boundary region near the wall by an infinitely thin film in three-velocity flow, is justified.

We write the equation describing the motion of the film and the change in the gas parameters on it for the stationary case, neglecting, as in /3,7/, the volume of the particles outside the film. Let $\mathbf{F}^\sigma$ denote the force acting on unit area of the film from the direction of the wall, and Q^σ the heat flux defined in the same manner. We direct the unit vector n

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**) Unfortunately, in the course of a further study of the separation conditions (see /8/ and :Gorbachev Iu.E. Boundary conditions in the theory of heterogeneous media. Boundary Layer. Leningrad, izd. FTI, 1980, preprint No.684), the use of an incorrect expression to describe the projection of the equation of motion of the film onto the normal to the wall made its separation within the framework of the model with zero intrinsic pressure of the particles impossible.

along the inner normal to the body and denote by F_n^σ the projection of $\mathbf{F}^\sigma$ on $\mathbf{n}$, putting $F_n^\sigma = -\mathbf{n}\mathbf{F}^\sigma$. Then $F_n^\sigma \geqslant 0$ on the film and the relation

$$F_n^\sigma = 0 \tag{1.1}$$

holds together with $\mathbf{F}_\tau^\sigma \equiv \mathbf{F}^\sigma + \mathbf{n}F_n^\sigma = 0$ and $Q^\sigma = 0$ on the free film, while on the attached film it can hold only in special cases (see below). If σ denotes an arbitrary area of the film and γ is its contour, then taking into account $\mathbf{F}^\sigma$ and Q^σ in Eqs.(3.3) of /3/ we arrive at the laws of conservation which hold on the free film ($\mathbf{F}^\sigma = 0, Q^\sigma = 0$) as well as on the attached film

$$\begin{gathered}
[\rho U_n] = 0, \quad [p + \rho U_n^2] + \rho_s^\sigma f_n^\sigma = 0, \quad (\rho U_n)_- [\mathbf{U}_\tau] + \rho_s^\sigma \mathbf{f}_\tau^\sigma = 0, \\
(\rho U_n)_- [I] + \rho_s^\sigma (\mathbf{U}_s^\sigma \mathbf{f}^\sigma + q^\sigma) = 0 \\
\iint_\sigma [\rho_s U_{sn}]\, d\sigma + \oint_\gamma \rho_s^\sigma U_{SN}^\sigma\, d\gamma = 0, \quad \iint_\sigma ([\rho_s U_{sn} \mathbf{U}_s] - \\
\rho_s^\sigma \mathbf{f}^\sigma - \mathbf{F}^\sigma)\, d\sigma + \oint_\gamma \rho_s^\sigma U_{sN}^\sigma \mathbf{U}_s^\sigma\, d\gamma = 0 \\
\iint_\sigma \{[\rho_s U_{sn} E_s] - \rho_s^\sigma (\mathbf{U}_s^\sigma \mathbf{f}^\sigma + q^\sigma) - Q^\sigma\}\, d\sigma + \\
\oint_\gamma \rho_s^\sigma U_{sn}^\sigma E_s^\sigma\, d\gamma = 0 \quad \left(E = e + \frac{1}{2} U^2,\ I = E + \frac{p}{\rho}\right)
\end{gathered} \tag{1.2}$$

Here $[\varphi] = \varphi_+ - \varphi_-$ is the difference in the values of φ on the film; the + and - signs are given to quantities with $n \to +0$ and $n \to -0$ respectively, n is measured in the direction of $\mathbf{n}$, the orientation of the unit vectors $\mathbf{n}$ at the points of the attached and free film is assumed to agree, p and e denote the pressure and specific (per unit mass) internal energy, $\mathbf{U}$, $U_n = \mathbf{n}\mathbf{U}$ and $\mathbf{U}_\tau = \mathbf{U} - \mathbf{n}U_n$ denote the velocity vector and its normal and tangential component to σ, $\mathbf{U}_{s\tau}^\sigma \equiv \mathbf{U}_s^\sigma$, since on the stationary film $\mathbf{n}\mathbf{U}_s^\sigma = 0$; $U = |\mathbf{U}|$; U_{sN}^σ is the projection of $\mathbf{U}_s^\sigma$ on the normal $\mathbf{N}$ to γ outward with respect to σ. The vector $\mathbf{N}$ as well as $\mathbf{U}_s^\sigma$ are tangent to the film, $\mathbf{f}^\sigma$ and q^σ denote the force and heat flux per unit area of the film acting from the direction of the gas, and are known functions of the scalar parameters of the film and gas (for a free film they act from both sides), and the differences $\mathbf{U}_\pm - \mathbf{U}_s^\sigma$. The only component of $\mathbf{F}^\sigma$ needed for the closure of (1.2) is the expression for $\mathbf{F}_\tau^\sigma$. The attached film clings to the wall, i.e. as we said before, we have

$$\mathbf{n}\mathbf{U}_s^\sigma = 0 \tag{1.3}$$

Therefore the projection of the equation of angular momentum of the film (the sixth equation of (1.2)) on the normal to itself determines, together with (1.3), F_n^σ for as long as $F_n^\sigma \geqslant 0$.

Let us stipulate that $\mathbf{f}^\sigma$, its components and q^σ vanish only when the difference in the corresponding velocities, their components and temperatures of the gas (T) and film (T_s^σ) all vanish. For $\mathbf{F}_\tau^\sigma$ we may, although not necessarily, adopt the law of "dry" friction according to which the force $\mathbf{F}_\tau^\sigma$ acts against $\mathbf{U}_s^\sigma$ and has a numerical value of KF_n^σ with $K > 0$.

In writing the equations of the film in the form (1.2) we assumed that the forces and heat fluxes acting over γ are small, although, in order to be able to consider the film as a whole (without appreciable stratification of the velocities and temperatures across its thickness), the forces acting within the film on any area normal to $\mathbf{n}$ and the corresponding heat fluxes, must be of the order of $F_n^\sigma, \mathbf{F}_\tau^\sigma$ and Q^σ. These demands do not contradict each other since in the presence of stresses and heat fluxes of the order of $F_n^\sigma, \mathbf{F}_\tau^\sigma$ and Q^σ the possibility of neglecting their contributions to the integrals over γ depends on the fact that the layer thickness is small. We note that in the stationary flow, $\mathbf{F}^\sigma$, having done no work above the film, affects only the redistribution of its kinetic and internal energy. For this reason the equation for the total energy of the film given in (1.2) does not contain $\mathbf{F}^\sigma$. In /7,8/ it is written incorrectly.

Even if condition (1.1) holds on the line Γ^* of separation between the film and the body, $\mathbf{F}_\tau^\sigma$ and Q^σ need not vanish on approaching Γ^* from the side of the attached film irrespective of the law of friction used. In the case of dry friction $\mathbf{F}_\tau^\sigma$ vanishes on Γ^* together with $\mathbf{F}_n^\sigma$. Equation (1.1) represents the condition of detachment only in the case when Γ^* differs from the line of break or discontinuity in the curvatures of the streamlined surface. We shall see below that here the film detaches from the body without forming a discontinuity in its curvatures. We shall call such a detachment "smooth". If Γ^* is the line of break or discontinuity in the surface curvature, then (1.1) does not, in general, hold when Γ^* is approached from the direction of the attached film. The limiting value $\mathbf{F}_n^\sigma > 0$, and a break in the curvature of the film occurs on Γ^*. Moreover, situations are possible in which the film, after parting from the body, makes finite angles on Γ^* with its parts attached to the body. Finally, instead of the film detaching from the body we can have a filament formed as a result of the intersection of the film trajectories and characterized by the finite linear density of the particles ρ^l in the general case of infinite ρ^σ, and even more so of ρ_s. The filament parameters will be denoted by the superscript l.

We will analyze the above possibilities by first deriving from (1.2) the equations describing the attached film up to its moment of separation. Since by virtue of the condition of impermeability we have $U_{n+} = 0$, at the wall, it follows that the first equation of (1.2) implies $U_{n-} = 0$, i.e. the gas does not diffuse into the film. This, together with (1.3), the remaining equations of (1.2) for the gas and the assumptions made about the character of the dependence of f_n^σ, $\mathbf{f}_\tau^\sigma$ and q^σ on the parameters of the medium, implies that

$$\mathbf{f}^\sigma = 0, \quad q^\sigma = 0 \tag{1.4}$$

and the velocity, as well as the temperature of the gas "inside" the attached film are identical with $\mathbf{U}_s^\sigma$ and T_s^σ, and, in general, differ from $\mathbf{U}_{\tau-}$ and T_-. The conclusions reached, which resemble those made reached in /7/ (where for some unknown reason the equality of the specific internal energies and not the temperatures is discussed), and agree with those reached in /3/ for films moving along the plane of symmetry of the flow are, on one hand, obvious, and on the other hand, largely conditional. Indeed, since we have zero gas flow through an attached film of zero thickness, it follows that the flow cannot act on a film with a finite particle flow rate; secondly, in such a situation we can talk of the properties of the gas in the film only by having in mind a transition to a layer of finite, though small, thickness. This corresponds completely to the situation existing for filaments /3/ where $\mathbf{f}^l = q^l = 0$, and within the cluster we have $\mathbf{U} = \mathbf{U}_s^l$ and $T = T_s^l$. Taking this into account, we shall delete the subscript s from parameters having the indices σ and l.

Taking into account (1.4), we will reduce the remaining equations of (1.2), which, together with (1.3) and the expressions for $\mathbf{F}_\tau^\sigma$ and Q^σ, describe the evolution of the attached film and define F_n^σ to the form

$$\begin{aligned}
&\iint_\sigma \rho_s U_{sn}\, d\sigma = \oint_\gamma \rho^\sigma U_N^\sigma\, d\gamma \\
&\iint_\sigma (\rho_s U_{sn} \mathbf{U}_s + \mathbf{F}^\sigma)\, d\sigma = \oint_\gamma \rho^\sigma U_N^\sigma \mathbf{U}^\sigma\, d\gamma \\
&\iint_\sigma (\rho_s U_{sn} E_s + Q^\sigma)\, d\sigma = \oint_\gamma \rho^\sigma U_N^\sigma E^\sigma\, d\gamma
\end{aligned} \tag{1.5}$$

The system contains, in addition to $\mathbf{F}^\sigma$ and $\mathbf{Q}^\sigma$, only the parameters of the particles precipitating onto the film (with the minus sign omitted), and of the film itself. The system can also be used for the segments on which the parameters are continuous, and for the filaments Γ^l representing the boundary lines and their discontinuities. The latter, as we said before, arise from the intersections of the film trajectories. At the segments of continuity system (1.5) is equivalent to the equations

$$\begin{aligned}
&\nabla_\sigma (\rho^\sigma \mathbf{U}^\sigma) - \rho_s U_{sn} = 0 \\
&\nabla_\sigma (\rho^\sigma U_k^\sigma \mathbf{U}^\sigma) - \rho_s U_{sn} U_{sk} - F_k^\sigma = 0 \quad (k = 1, 2, 3) \\
&\nabla_\sigma (\rho^\sigma E^\sigma \mathbf{U}^\sigma) - \rho_s U_{sn} E_s - Q^\sigma = 0
\end{aligned} \tag{1.6}$$

Here U_k^σ and F_k^σ are the projections of $\mathbf{U}^\sigma$ and $\mathbf{F}^\sigma$ onto the axes of a fixed, rectilinear $x_1x_2x_3$ system of coordinates which can be conveniently attached to the point of the film under consideration and orientate it so that its unit vectors $\mathbf{i}_1$, $\mathbf{i}_2 = \mathbf{n}$ and $\mathbf{i}_3$ form a right-handed triad and $\mathbf{i}_1$ is directed along $\mathbf{U}^\sigma$. The change from (1.5) to (1.6) is based on the use of Green's formula for smooth surfaces /3/

$$\oint_\gamma A_N^\sigma d\gamma = \iint_\sigma \nabla_\sigma \mathbf{A}^\sigma d\sigma$$

where the surface divergence acting on the continuous vector $\mathbf{A}^\sigma$, tangential to σ, is defined by the equation (γ is contracted to a point)

$$\nabla_\sigma \mathbf{A}^\sigma = \lim_{\sigma \to 0} \frac{1}{\sigma} \oint_\gamma A_N^\sigma d\gamma$$

Let ϑ be the angle between the x_1-axis and the projection of the film streamline onto the plane x_1x_3, tangential to the film, and let the positive rotation of ϑ correspond to anticlockwise rotation of $\mathbf{i}_1$ (from the side of the body) to its coincidence with $\mathbf{U}^\sigma$. Then we can show that near the point in question $U_1^\sigma = U^\sigma + O(\vartheta^2)$, $U_3^\sigma = U^\sigma\vartheta + O(\vartheta^3)$, and the operator Δ_σ on the surface should be treated in the same manner as the operator Δ on the plane. In particular, for any scalar χ we have

$$\nabla_\sigma(\chi\mathbf{U}^\sigma) = \mathbf{U}^\sigma\nabla_\sigma\chi + \chi\nabla_\sigma\mathbf{U}^\sigma$$
$$\left(\nabla_\sigma\chi = \mathbf{i}_1\frac{\partial\chi}{\partial x_1} + \mathbf{i}_3\frac{\partial\chi}{\partial x_3},\ \nabla_\sigma\mathbf{U}^\sigma = \frac{\partial U^\sigma}{\partial x_1} + U^\sigma\frac{\partial\vartheta}{\partial x_3}\right)$$

where $\Delta_\sigma\chi$ is the gradient at the film surface.

Let us introduce the angle θ defining the deviation of $\mathbf{U}^\sigma$ from the tangent x_1x_3-plane. Let θ be positive for $U_2^\sigma = \mathbf{i}_2\mathbf{U}^\sigma > 0$. We stress that $\mathbf{i}_2 = \mathbf{n}$ and $U_2^\sigma = U_n^\sigma$ only at the origin of coordinates of the reference system chosen. Then $U_2^\sigma = U^\sigma\theta + O(\theta^2)$, and we transform system (1.6), with due regard to what has been already said, to the form

$$U^\sigma\frac{\partial\rho^\sigma}{\partial\gamma} + \rho^\sigma\left(\frac{\partial U^\sigma}{\partial\gamma} + U^\sigma\frac{\partial\vartheta}{\partial N}\right) = \rho_s U_{sn} \tag{1.7}$$
$$\rho^\sigma U^\sigma\frac{\partial U^\sigma}{\partial\gamma} = \rho_s U_{sn}(U_{s\gamma} - U^\sigma) + F_\gamma^\sigma$$
$$\rho^\sigma(U^\sigma)^2\frac{\partial\vartheta}{\partial\gamma} = \rho_s U_{sn}U_{sN} + F_N^\sigma$$
$$\rho^\sigma U^\sigma\frac{\partial E^\sigma}{\partial\gamma} = \rho_s U_{sn}(E_s - E^\sigma) + Q^\sigma$$
$$F_n^\sigma = \rho_s U_{sn}^2 - \rho^\sigma(U^\sigma)^2 k_n^\sigma$$

Here $\partial/\partial\gamma \equiv \partial/\partial x_1$ and $\partial/\partial N \equiv \partial/\partial x_3$ are derivatives along and normal to the streamline

$$U_{s\gamma} \equiv U_{s1},\ U_{sN} \equiv U_{s3},\ F_\gamma^\sigma \equiv F_1^\sigma$$

and $F_N^\sigma \equiv F_3^\sigma$, and the fact that $F_2^\sigma = -F_n^\sigma$, $k_n^\sigma = \partial\theta/\partial\gamma$ represents the curvature of the normal section of the surface containing the streamlines ($k_n^\sigma > 0$, provided that the body is convex in the direction of the stream) is taken into account. The angles ϑ and θ in (1.7), the change in which characterizes bending of the film streamlines in the tangential and normal plane, are measured from arbitrarily chosen directions.

The equation for F_n^σ from (1.7) follows from the second equation of (1.6), with $k = 2$. It is written last, since it serves to determine F_n^σ in terms of k_n^σ, and the film and flow parameters obtained by solving the first four equations (or three when F_γ^σ and F_N^σ are independent of T^σ) of (1.7) supplemented by the expressions for $F_\gamma^\sigma, F_N^\sigma$ and Q^σ. Clearly, if F_n^σ appears in these expressions, as in the case of dry friction when $F_N^\sigma \equiv 0$, and $F_\gamma^\sigma = -KF_n^\sigma$, then the last equation is used to eliminate F_n^σ from them. The second, third and fourth equations of (1.7) are in their characteristic form. Taking into account the equation for F_n^σ, we reduce the condition of smooth detachment (1.1) to the form

$$\rho_s U_{sn}^2 - \rho^\sigma(U^\sigma)^2 k_n^\sigma = 0 \text{ on } \Gamma^s \tag{1.8}$$

Unlike the expresion for F_n^σ in (1.7), that given in /8/ lacks the term containing k_n^σ, due to an error. In spite of this, Gorbachev /8/ connected the detachment with the "supersonic" nature of the flow within the film and wrote, initially, (see note on p.507) the condition of detachment in the form (1.8). This formulation however is not mentioned in /8/.

Equations describing the evolution of the filaments which are formed on the streamline surfaces as a result of intersections of the film streamlines, can also be obtained from (1.5). If we measure the distance Γ along the filament (in the direction of motion of its particles) assign the indices Γ and N to the projections of the corresponding vectors onto the tangent and normal to the filament, and introduce the angles ϑ and θ for the filament in the same manner as for the film streamlines, then the equations sought will become

$$d(\rho^l U^l)/d\Gamma = j_1 + j_2,\quad \rho^l U^l dU^l/d\Gamma = j_1(U_{\Gamma 1}^\sigma - U^l) + \tag{1.9}$$
$$j_2(U_{\Gamma 2}^\sigma - U^l) + F_\Gamma^l,\quad \rho^l(U^l)^2 d\vartheta/d\Gamma = j_1 U_{N1}^\sigma + j_2 U_{N2}^\sigma + F_N^l$$
$$\rho^l U^l dE^l/d\Gamma = j_1(E_1^\sigma - E^l) + j_2(E_2^\sigma - E^l) + Q^l$$
$$-\rho^l(U^l)^2 k_n^l = F_n^l$$

Here $j = \rho^\sigma|U_N^\sigma|$ is the flux density of film particles arriving at unit length of the filament (from each of its sides), the indices 1 and 2 denote the film parameters to the left and right of the filter relative to the direction of motion of the particles within it, $k_n^l = d\theta/d\Gamma$ is the curvature of the normal section of the surface in the direction of the cluster, $F_\Gamma^l, \ldots$ characterize the action of the wall on unit length of the cluster and are brought in the same manner as $F_\gamma^\sigma, \ldots$, and $F_n^l \geqslant 0$. We assume that in the model in question /3/ the particles do not pass from the film to the mixture, nor from the cluster to the film. Within this approximation we have $U_{N2}^\sigma \leqslant 0$, while $U_{N1}^\sigma \geqslant 0$. The modifications needed for the cluster moving along the fold in the streamline surface will be discussed below.

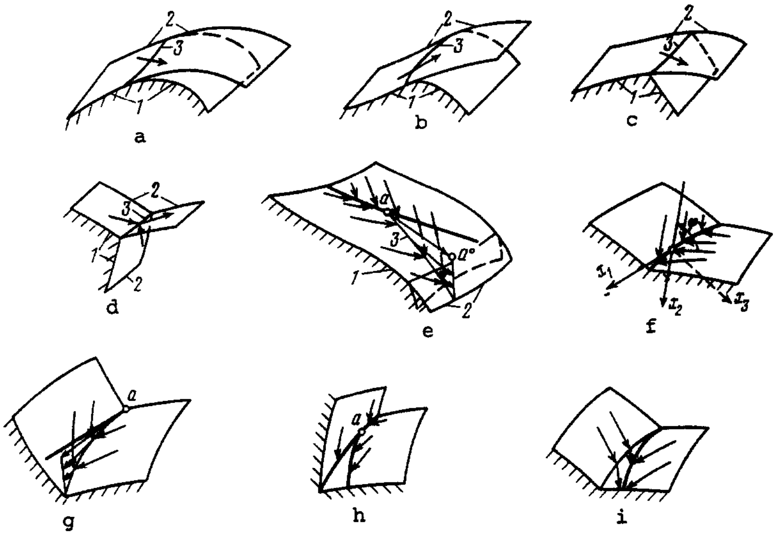

Fig.1

2. Using the equations and conditions derived above, we will investigate various ways in which the film may become detached from the body, as well as the formation and detachment of filaments. We will begin with a smooth detachment for which the curvature k_n^σ is continuous on the line of detachment along the film streamline. Since for the attached film we have $F_n^\sigma \geqslant 0$, from the last equation of (1.7) it follows that the condition of smooth detachment (1.8) can hold only when $k_n^\sigma > 0$, i.e. if the normal cross section of the body is convex towards the flow. Such a detachment occurs when k_n^σ increases fairly rapidly (Fig.1a, numerals 1,2 and 3 denote the body, the film and the line of detachment respectively, and the arrow indicates the direction of motion of the particles). We shall show that in this case k_n^σ on Γ^s is continuous for the detached film. To do this we will write the equations of the free stationary film, which by virtue of (1.2) have the form

$$
\begin{aligned}
&U^\sigma \partial\rho^\sigma/\partial\gamma + \rho^\sigma(\partial U^\sigma/\partial\gamma + U^\sigma\partial\vartheta/\partial N) = -[\rho_s U_{sn}] \\
&\rho^\sigma U^\sigma \partial U^\sigma/\partial\gamma = -[\rho_s U_{sn}(U_{s\gamma} - U^\sigma)] + \rho^\sigma f_\gamma^\sigma \\
&\rho^\sigma (U^\sigma)^2 \partial\vartheta/\partial\gamma = -[\rho_s U_{sn} U_{sN}] + \rho^\sigma f_N^\sigma \\
&\rho^\sigma (U^\sigma)^2 \partial\theta/\partial\gamma = -[\rho_s U_{sn}^2] + \rho^\sigma f_n^\sigma \\
&\rho^\sigma U^\sigma \partial E^\sigma/\partial\gamma = -[\rho_s U_{sn}(E_s - E^\sigma)] + \rho^\sigma(\mathbf{U}^\sigma \mathbf{f}^\sigma + q^\sigma)
\end{aligned}
\tag{2.1}
$$

The particles precipitate on the film detached from the smooth body (without a break in Γ^s, although possibly with a discontinuity in k_n^σ). From one side only. Therefore we have, on the film or at least near Γ^s, $[U_{sn}\varphi] = -(U_{sn}\varphi)_- \equiv -U_{sn}\varphi$, where φ denotes any parameter and the last step consists simply of omitting the minus subscript. In addition, when the line of smooth detachment is approached from the free-film side, $U_{n-} \to U_{n+} \to 0$, and, as was shown before, $\mathbf{f}^\sigma$ and q^σ in the right sides of (2.1) vanish. This, together with the penultimate equation of (2.1), shows that (1.8) holds on the film immediately after its smooth detachment. Since $k_n^\sigma = \partial\theta/\partial\gamma$, this ensures that k_n^σ remains continuous during the passage through Γ^s.

Let us now suppose that k_n^σ has a break on Γ^s, increasing at the same time (e.g. from $k_{n-}^\sigma < 0$ to $k_{n+}^\sigma > 0$, where the minus and plus signs denote the limiting values before and after detachment. It may happen that when $F_{n-}^\sigma > 0$, the value of F_{n+}^σ found from the last equation of (1.7) becomes negative. This means that the film will detach itself from the body on Γ^s and its curvature will increase with a jump, without however reaching the corresponding value of the body. In fact, repeating the previous discussion for this case, we can show that if the curvature of the film and the body are the same up to Γ^s (this is of course obligatory for the attached film) then, after the detachment k_n^σ can be found from condition (1.8) where all the parameters except k_n^σ are continuous. The situation is illustrated schematically in Fig.1b. The limit of the case just discussed is represented by the separation from the salient line (Fig.1c). Here, as in the previous case, the film detached from the body has a discontinuity in curvature on Γ^s (with continuous θ). Now we obviously have $U_{n+} \neq 0$, therefore in determining $k_n^\sigma = (\partial\theta / \partial\gamma)_+$ from the penultimate equation of (2.1), we must take into consideration the term containing f_n^σ. Because of this k_{n+}^σ does not satisfy condition (1.8). We shall call the last two types of detachment "almost smooth".

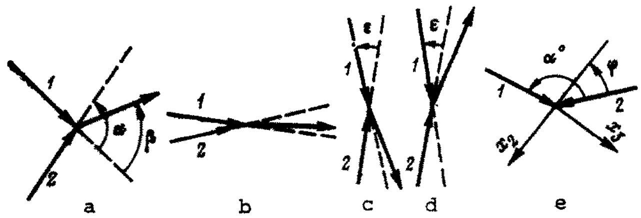

Fig.2

"Nonsmooth" separations with a break in the film along Γ^s can have various causes. In the simplest case two attached films meet, approaching the sharp bend of the body from opposite sides (Fig.1d). In this case we need additional assumptions regarding the interaction (collision) of the films with each other and with the wall in the neighbourhood of the bend. We assume that the colliding films depart as a single film of uniform thickness, i.e., that the differences in the parameters even out rapidly and the body exerts no concentrated force on Γ^s.

Now consider the plane Σ normal to Γ^s, where α is the angle between the upper and lower generatrix of the body and β is the angle between the generatrix of the film and the upper generatrix of the body (Fig.2a). If V^σ and W^σ are the projections of U^σ on Σ and Γ^s, then the angle β and other parameters of the film detached from the body are given by the relations which follow from the laws of conservation of mass, momentum and energy

$$\begin{gathered} \operatorname{tg}\beta = (\chi + \cos\alpha)^{-1}\sin\alpha, \quad \rho^\sigma = (A^m)^2/A^\tau, \quad V^\sigma = A^\tau/A^m \\ W^\sigma = A^n/A^m, \quad E^\sigma = A^e/A^m, \quad \chi = \rho_1^\sigma (V_1^\sigma)^2/(\rho_2^\sigma (V_2^\sigma)^2) \\ A^m = \rho_1^\sigma V_1^\sigma + \rho_2^\sigma V_2^\sigma, \quad A^\tau = \rho_1^\sigma (V_1^\sigma)^2 \cos\beta + \\ + \rho_2^\sigma (V_2^\sigma)^2 \cos(\alpha - \beta), \quad A^n = \rho_1^\sigma V_1^\sigma W_1^\sigma + \rho_2^\sigma V_2^\sigma W_2^\sigma \\ A^e = \rho_1^\sigma V_1^\sigma E_1^\sigma + \rho_2^\sigma V_2^\sigma E_2^\sigma \end{gathered} \tag{2.2}$$

Here, as in Fig.2, numerals 1 and 2 denote the upper and lower attached film and the parameters without indices refer to the free film. All quantities are taken on Γ^s. Naturally, if the flux of the component of momentum normal to Γ^s of one of the films becomes vanishingly small compared with the same characteristics of the other film, then β will tend, according to (2.2), either to zero or to α, i.e., a continuous transformation to almost smooth detachment will be observed. Further, by virtue of the first formula of (2.2) $0 \leqslant \beta \leqslant \alpha$, and for any fixed $\alpha < \pi$, $\beta \to \alpha / 2$, as $\chi \to 1$ which follows at once from symmetry considerations. For small angles of collision $\alpha \ll 1$ (Fig.2b) we have $\beta \approx \alpha / (1 + \chi)$ and for $\alpha = \pi - \varepsilon$ with positive $\varepsilon \ll 1$ (Fig.2c and d) we have

$$\beta \approx \begin{cases} \varepsilon/(\chi - 1) & \text{when} \quad \chi > 1 \\ \pi - \varepsilon/(1 - \chi) & \text{when} \quad \chi < 1 \end{cases} \tag{2.3}$$

When the sharp bend in the body disappears, $(\varepsilon = 0)$, the resulting film when $\chi \neq 1$ by virtue of (2.3), comes into contact with one of the attached films on Γ^s. Two possibilities must be considered here. If the contact occurs only along Γ^s, then Γ^s must become the line of almost smooth detachment. Such a situation could occur as a result of change in the orientation of the streamlines of the departing film if this resulted in the value of the curvature k_n^σ becoming larger than k_{n1}^σ and k_{n2}^σ, provided that the curvature of the streamlined surface were continuous. If however we take into account the fact that in this case one of the attached films (say the upper) is screened near Γ^s from the precipitating particles by the detached film, then the situation discussed can only occur (naturally, only in principle) on a body whose principal curvatures are of different signs. Otherwise, with $k_{n1}^\sigma > 0$, the screening will cause the upper film to become detached without reaching Γ^s. In particular, if any section of the body near Γ^s is convex towards the gas, then separation with tangential contact (in the Σ plane) on the line of collision of attached films is, in general, impossible.

Before explaining what can be expected in such such situations, we shall consider the second possibility, when the film detached from the body along the tangent, spreads over it together with one of the initial attached films. The simultaneous presence on the surface of two, mutually permeating films, cannot be justified, neither by the condition for the two-fluid model to be valid (one-fluid for the particles), nor by reasons of conformity with the actual flow pattern. The introduction of filaments absorbing the particles from colliding films offers a natural way out of this difficulty. When the mean free paths λ^σ of the particles of the mutually penetrating films are short ($\lambda^\sigma < \lambda$), the filaments represent a rational schematic representation of the narrow zones in which the particle density is even greater than in the boundary layers. We note, by the way, that because of the screening effect only a single free film can originate at the sharp corner, with initial parameters given by (2.2), and not two films which we would expect at first glance (one film generated by the attached films coming into contact with each other, and the other resulting from the intersection of the particle trajectories streamlining the body from different sides).

Th cluster appearing at the smooth surface as a result of the intersection of the film streamlines with $\rho^\sigma < \infty$, is described by the system (1.9). The same system but without the right sides describes the free filaments which, by virtue of this fact, move along rectilinear trajectories with constant parameters. The latter can be easily understood by recalling that, because of the zero thickness, the interaction between the filament and the gas is unimportant, and there are assumed to be no mass forces proportional to ρ^l in the present case. For a filament moving along the surface of the body and separating the attached films (by absorbing them within itself) with velocity vectors pointing towards the filament, the componet of the force $F_n^{\,l}$ normal to the wall is non-negative. For this reason the filament will remain on the surface only for as long as $k_n^{\,l} < 0$ (Fig.1e, the filament is shown by the thick line). At point a at which $k_n^{\,l}$ changes its sign, the filament leaves the surface while in tangential contact with it.

Since the trajectories of the film particles continue to intersect even after the departure of the filament, a new filament forms at the surface of the body the trace of which will, in general, make a finite angle with the previous filament. The initial parameters of the new filament (including its direction) are found from the solution of the problem dealing with collision, on a smooth surface, of two attached films with different constant parameters. Let us make the quite natural assumption that the force $\mathbf{F}^l$ and heat flux Q^l arriving at unit length of the regenerated cluster, also vanish at the initial point where $\rho^l = 0$. Then the problem in question becomes selfsimilar and it can be solved within the framework of (1.9), with $F_\Gamma^{\,l} = F_N^{\,l} = Q^l = 0$, in the same manner as the analogous problem of the appearance of a film /3/. The orientation of the filament at the point a of its appearance on the surface is given by the equation

$$\chi \equiv \rho_1^\sigma (U_{N1}^\sigma)^2/\{\rho_2^\sigma (U_{N2}^\sigma)^2\} \equiv \rho_1^\sigma (V_1^\sigma)^2/\{\rho_2^\sigma (V_2^\sigma)^2\} = 1 \tag{2.4}$$

while the cluster density ρ^l near a is a linear function of the distance Γ from a, i.e. $\rho^l = R\Gamma$, and its remaining characteristics are close to the constants which are given, together with R, by the equations

$$R = (j_1 + j_2)^2/(j_1 U_{\Gamma 1}^\sigma + j_2 U_{\Gamma 2}^\sigma), \quad U^l = (j_1 U_{\Gamma 1}^\sigma + j_2 U_{\Gamma 2}^\sigma)/(j_1 + j_2) \tag{2.5}$$
$$E^l = (j_1 E_1^\sigma + j_2 E_2^\sigma)/(j_1 + j_2)$$

where, as in (2.4), all values are taken at a.

Let $k_n^{\,l} \leqslant 0$ in the direction defined by (2.4). Then the filament will remain attached to the wall up to the point at which $k_n^{\,l}$ changes its sign, and its evolution will be described by system (1.9) with intitial values (2.5). Otherwise, if we already have $k_n^{\,l} > 0$, at the starting point, then the filament leaves the body at the instant of its conception and generates a free film. Since in this case a filament with $\rho^l = 0$ appears and leaves the body at once at every point of Γ^s, it follows that condition (2.4) holds everywhere on Γ^s. Therefore in accordance with the analysis carried out above for the collision of films on a smooth surface, the free film leaves the body making with it a finite angle (in the Σ plane) with its streamlines touching Γ^s (Fig.1e). Generally the upper edge of the film aa° does not coincide with the filament which has arrived at a, since the initial orientations are different, and aa° together with the film becomes curved, unlike the filament, during the interaction with the mixture. System (2.1) with initial conditions on Γ^s where $U^\sigma = U^l$ and $E^\sigma = E^l$ with U^l and E^l from (2.5), describes the evolution of the detached film. However, since $\mathbf{U}^\sigma$ touches Γ^s, therefore, unlike the case of detachment from the edge (Figs.1d and 2), in determining ρ^σ on Γ^s and the initial inclination of the departing film to the surface Σ we take into account its interaction with the mixture and the fact that the Γ^s becomes bent on the surface of the body. Without spending more time on this problem, we shall show that if $k_n^{\,l} \equiv k_n^{\Gamma^s} > 0$ already at the initial point of intersection of the attached film streamlines, then the mechanism of formation and detachment of the film comes into operation at once without the formation of an attached filament. Here ρ^σ tends to zero as it approaches aa°

If the boundary cluster flows around the corner (edge) with $\alpha > \pi$, then in order to derive the equations describing its evolution it is convenient to use any Cartesian coordinates, the x_1x_2-plane of which represents one of the planes belonging to the bundle passing through the tangent to the edge (the x_1-axis points towards $\mathbf{U}^l$ along the edge, and x_2 towards the inside of the body). In any such system the equations of motion of the filament will reduce to the first, second and fourth equation of (1.9) defining ρ^l, U^l and E^l. Moreover, we will find the following relation must hold for all φ in the range $0 \leqslant \varphi \leqslant \alpha^\circ = 2\pi - \alpha < \pi$

$$\Phi(\Gamma, \varphi) \equiv \rho_1^\sigma (V_1^\sigma)^2 \cos(\alpha^\circ - \varphi) + \rho_2^\sigma (V_2^\sigma)^2 \cos\varphi - \rho^l (U^l)^2 k_\varphi = F_\varphi{}^l \geqslant 0 \tag{2.6}$$

Here the angle φ is introduced in the manner shown in Fig.1f and 2e, k_φ is the curvature of the projection of the edge on the x_1x_2-plane and $k_\varphi > 0$, if this projection is convex towards the gas, and $F_\varphi{}^l$ is the projection of the reaction of the body against the filament in the negative direction of the x_2-axis. The condition under which the motion in question can be realized is (this is an additional assumption), that the quantity $F_\varphi{}^l$ obtained form (2.6) should be non-negative for all $0 \leqslant \varphi \leqslant \alpha^\circ$. The change in the sign of Φ at some point when $\varphi = \varphi^*(\Gamma_a)$ results in the filament either joining the flow, or moving to one of the walls. When the filament joins the flow, which as we well know takes place when $0 < \varphi^* < \alpha^\circ$, a new filament may begin to form at the edge, with the initial parameters given by (2.5) and with $\rho^l = R(\Gamma - \Gamma_a)$ near a. The inequality from (2.6) may hold for this filament, e.g. because of the decrease (down to zero at the point a) of the third term on the left-hand side. If on the other hand the inequality does not hold when $\rho^l = 0$, after the filament joins the flow, a free film begins to emerge from the edge, the particles of which leave the body while touching the edge (Fig.1g). The angle $\varphi \equiv \varphi^\sigma$ of the departing film is given by the equation $\operatorname{ctg}\varphi^\sigma = (1 + \chi\cos\alpha^\circ) / (\chi\sin\alpha^\circ)$.

If $\varphi^* = 0$ or α° and $k_n{}^l < 0$ on the line of contact between the filament which has left the edge, and the corresponding edge of the body, then the filament will begin to move further along the body and screen the edge from one of the initial attached films (Fig.1h). The flow near the edge will differ when $\alpha^\circ < \pi/2$ from the one discussed only in the absence of the screened film. In contrast to this, we find that when $\pi/2 < \alpha^\circ < \pi$, then another, fundamentally different situation becomes possible in which the particles belonging to one of the films (we shall assume, to be specific, that it is the left one as shown in Fig.1i) will cross the edge and pass to the other edge. Here a concentrated force $F_{n2} = \rho_1^\sigma (V_1^\sigma)^2 \sin\alpha^\circ$, acts on the body along the normal to the second edge, over unit length of the edge, and the initial parameters of the film at the edge are

$$\rho^\sigma = \rho_1^\sigma/\cos\alpha^\circ, \quad V^\sigma = V_1^\sigma\cos\alpha^\circ, \quad W^\sigma = W_1^\sigma, \quad E^\sigma = E_1^\sigma$$

3. Fig.3 shows the patterns of two-dimensional flows with films and filaments. The films and filaments are drawn in thick lines, the gas (particle) streamlines in thin continuous (dashed) lines, and the shock waves (the c^+-characteristics) with double dotted lines. The body contours are cross hatched. Figs.3a and b depict the flows realized within the framework of the model discussed, in the Laval nozzle where the film is formed from the particles arriving at the narrowing walls and must leave the wall near the smallest cross section. If this did not occur, then F_n^σ in (1.7) would become negative since here we have $k_n^\sigma > 0$ and $\rho_s = 0$ by virtue of the appearance of a boundary zone of pure gas. Further evolution of the flow depends on the magnitude (and direction) of the momentum of the film $\mathbf{I}^\sigma \equiv \rho^\sigma U^\sigma \mathbf{U}^\sigma$ at the point of separation. If the impulse $\mathbf{I}^\sigma$ is not excessively large, then the film through which a stream of mixture passes, filtering, as it were, the particles off during its passage, causes the film to change it direction of motion and reach the wall, while the separating the zones containing the mixture and the pure gas. After making contact with the wall and losing the normal component of the momentum, the film becomes reattached (Fig.3a).

If the vertical component of $\mathbf{I}^\sigma$ is large at the instant of separation, which occurs in the case of channels with fairly curved walls and large contraction, then the film attains the plane or axis of symmetry and forms there either a film, or a cluster (we assume that only a single "stream" of particles appears as a result of merging). Such a situtation is shown in Fig.3b. Without writing out the formulas for determining the parameters of a similar stream, we note that in the plane case it reduces to (2.2) with $\chi = 1$. For a converging axisymmetric film we find that because of tending to infinity $\rho^\sigma \sim y^{-1}$, where y is the distance from the axis of symmetry, it is advisable, when deriving the formulas, to take the mass flux of the film at y as small compared with the relaxation length, but nevertheless finite.

Let us now explain the behaviour of the film parameters near the critical point o of a blunt body, where we also place the origin of Cartesian or cylindrical coordinates (Fig.3c). Let u and v be the x-and y-velocity components; $\nu = 0$ and 1 in the plane and axisymmetric case. A film or filament not shown in Fig.3c may be present in the oncoming flow on the x-axis. Then near o we have

$$\rho^\sigma(y) \approx \frac{(\rho_s u_s y^{1+\nu} + \delta)^2}{(1+\nu)^2 \rho_s u_s y^\nu J(y)}, \quad U^\sigma(y) \equiv v^\sigma(y) \approx \frac{(1+\nu)\rho_s u_s J(y)}{\rho_s u_s y^{1+\nu} + \delta} \tag{3.1}$$

$$J(y) = \int_0^y v_s(\eta)\eta^\nu d\eta, \quad \delta = \begin{cases} \rho_\infty^\sigma \dot{U}_\infty^\sigma & \text{when} \quad \nu = 0 \\ \rho_\infty^l U_\infty^l & \text{when} \quad \nu = 1 \end{cases}$$

and $E^\sigma(0)$ is equal to $E_\infty{}^\sigma$ or $E_\infty{}^l$ when $\delta \neq 0$ and to $E_s(0)$ when $\delta = 0$. In (3.1) ρ_s and u_s are taken at $y = 0$; $v_s(0) = 0$ and $\delta \neq 0$, if a film or filament arrives at o, and their parameter are denoted by the subscript ∞. Since $J(y) = o(y)$ for any $v_s(y) = O(y)$, therefore $\rho^\sigma(y) \to \infty$ when $\delta \neq 0$. At the same time we have

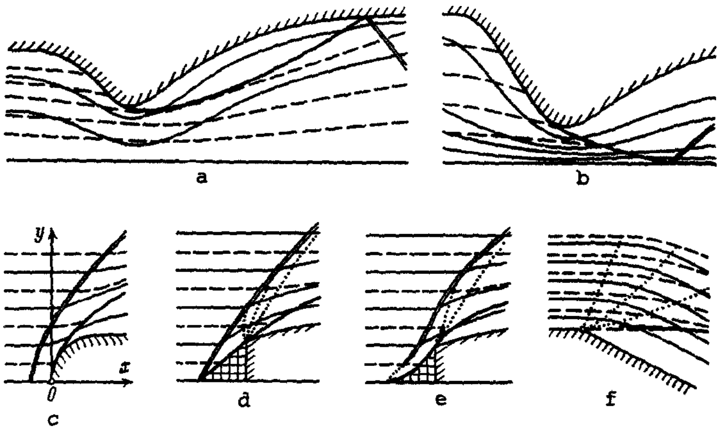

Fig.3

$$\lim_{y \to 0} \int_0^y \rho^\sigma(\eta)\eta^\nu d\eta = \infty \tag{3.2}$$

Here $\rho^\sigma(0)$ is unbounded also when $\delta = 0$, provided that $v_s(y)/y = o(y)$ and for (3.2) it is sufficient that $v_s(y)/y^{2+\nu} = o(y)$. The unboundedness of $\rho^\sigma(0)$ and (3.2) follows from the fact that in the model in question $U^\sigma \equiv v^\sigma$ is generated by the y-component of the momentum of the particles arriving at the film, and the component is much too small in the cases described above to accelerate the film sufficiently rapidly. We note in this connection that if $\delta = 0$, then the quantity $\rho^\sigma(0)$ is finite when $v_s(y)/y = O(1)$ and equal to zero when $y/v_s(y) = o(y)$.

One of the possible methods of constructing a model preventing undesirable situations, with ρ^σ increasing without limit and with (3.2), consists of introducing the pressure of the "gas" composed of the particles. Without discussing the details of such an approach, we shall indicate an alternative which seems to be at least as plausible under certain conditions. It is illustrated by Fig.3d ($\delta = 0$) and 3e ($\delta \neq 0$), where the cross hatching indicates a "ridge", i.e. a zone containing loose particles of finite volume. When $\delta \neq 0$, the angle at the top of the ridge is zero because the loose medium cannot resist a concentrated force which would otherwise appear at the point at which the arriving film or filament breaks up. The complete solution of the problem with a ridge assumes the use of the theory of the filtration of a gas through a moving particulate medium. The problem is simplified if the momentum fluxes of the gas and particles are small compared with the stresses of the particulate medium at the ridge, because the motion of the particulate medium at the ridge filtering the gas can then be neglected. Without writing put the equations of motion of the film along the ridge, we recall that in this case, unlike the previous case, $U_n \neq 0$ at the boundary separating the ridge from the surface of discontinuity of the porosity /9/.

Finally we stress that no such features appear in flows past sharp bodies and the introduction of the ridges is not, in general, necessary, including the case of a filament impinging on an axisymmetric body when $\rho^\sigma(0) \neq \infty$. In the latter case, which is the converse of the film problem of a film focusing on the axis (Fig.3c), unlike (3.1) $\rho^\sigma(y) \sim y^{-1}$, and the left side of (3.2) is equal to zero. In the remaining cases of flows past sharp bodies we have $\rho^\sigma(0) = 0$, if $\delta = 0$ and $\nu = 0$ and 1, and $\rho^\sigma(0)$ is finite when $\delta \neq 0$ and $\nu = 0$.

Fig.3f explains the appearance of a film in a supersonic flow, initially uniform and in equilibrium, past a convex angle. In a strong rarefaction bundle which appears in such a flow caused by a drop in p and T, the density and viscosity of the gas as well as the factor

φ_f appearing in the expression for the force of interaction between the phases $f = \varphi_f \cdot (U - U_s)$, can decrease so strongly that the line of relaxation of the particles with respect to velocity becomes many times greater behind the bundle than in front of it. As a result, the particles moving at various distances from the wall diverge in the initial part of the bundle by an angle which is greater the farther from the wall the particle intersects the bundle, and will afterwards move almost along straight lines, invariably intersecting and forming a film.

In conclusion, we note that the analysis carried out above is based on the idea of representing films and filaments as surfaces and lines. Actually, the volume density of the second phase cannot be greater than that of the "dense" packing of the particles $\rho_m = \gamma\rho_s^\circ$ where ρ_s° is the density of the particle material and the coefficient $\gamma < 1$ depends on the form of the particles. This makes it possible to estimate, in the solution constructed, the minimum possible thicknesses of the films as ρ^σ/ρ_m, and the radii of the filaments as $\sqrt{\rho^l/(\pi\rho_m)}$, and then to correct e.g. the trajectory of the cluster acted upon by the gas and particle flows. The mechanisms of diffusion connected with removal of particles from films by gas streams passing through them also calls for additional investigation. Nevertheless, if the mean free path of the dispersed phase is noticeably less than the characteristic dimension of the problem, then even without taking diffusion into account, the solutions with films and filaments yield qualitatively (and at times quantitatively) correct results. Thus, if the solution shown in Fig.3a(b) is realized in a flow through a nozzle, then the maximum density of the particles should be expected at its wall (axis). Finally, we point out how close the problem discussed above are to those appearing in hypersonic gas dynamics in connection with formation of compressed layers in the Newton-Buseman model approximation. In the latter case the condition of detachment of the layer is that the gas pressure becomes equal to zero on one of its sides.

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